Selected Solutions for Chapter 7: Quicksort

Solution to Exercise 7.2-3

PARTITION does a "worst-case partitioning" when the elements are in decreasing order. It reduces the size of the subarray under consideration by only 1 at each step, which we've seen has running time $\Theta(n^2)$.

In particular, PARTITION, given a subarray $A[p \mathinner{..} r]$ of distinct elements in decreasing order, produces an empty partition in $A[p \mathinner{..} q-1]$, puts the pivot (originally in $A[r]$) into $A[p]$, and produces a partition $A[p+1 \mathinner{..} r]$ with only one fewer element than $A[p \mathinner{..} r]$. The recurrence for QUICKSORT becomes $T(n) = T(n-1) + \Theta(n)$, which has the solution $T(n) = \Theta(n^2)$.

Solution to Exercise 7.2-5

The minimum depth follows a path that always takes the smaller part of the partition—i.e., that multiplies the number of elements by α. One iteration reduces the number of elements from n to αn, and i iterations reduces the number of elements to $\alpha^i n$. At a leaf, there is just one remaining element, and so at a minimum-depth leaf of depth m, we have $\alpha^m n = 1$. Thus, $\alpha^m = 1/n$. Taking logs, we get $m \lg \alpha = -\lg n$, or $m = -\lg n / \lg \alpha$.

Similarly, maximum depth corresponds to always taking the larger part of the partition, i.e., keeping a fraction $1 - \alpha$ of the elements each time. The maximum depth M is reached when there is one element left, that is, when $(1-\alpha)^M n = 1$. Thus, $M = -\lg n / \lg(1-\alpha)$.

All these equations are approximate because we are ignoring floors and ceilings.